1. Given the following functions, determine $f'(x)$:

   (a) $f(x) = x^3 e^x$

   $f = x^3 \quad g = e^x$

   $f' = 3x^2 \quad g' = e^x$

   $f'(x) = f'g + fg' = 3x^2 \cdot e^x + x^3 \cdot e^x$

   (b) $f(x) = \sqrt{x} \ln x$

   $f = x^{1/2} \quad g = \ln x$

   $f' = \frac{1}{2} x^{-1/2} \quad g' = \frac{1}{x}$

   $f'(x) = f'g + fg' = \frac{1}{2} x^{-1/2} \cdot \ln(x) + x^{1/2} \frac{1}{x}$

   $= \frac{\ln(x)}{2\sqrt{x}} + \frac{1}{\sqrt{x}}$

   (c) $f(x) = (x^3 - 3\ln x)(2e^x + 3x)$

   $f = x^3 - 3\ln x \quad g = 2e^x + 3x$

   $f' = 3x^2 - \frac{3}{x} \quad g' = 2e^x + 3$

   $f'(x) = f'g + fg' = \left(3x^2 - \frac{3}{x}\right)(2e^x + 3x) + (x^3 - 3\ln x)(2e^x + 3)$

   (d) $f(x) = (3e^x + \frac{1}{x})(1 + \frac{1}{x^2})$ (*Hint*: Write everything with variables in the numerator.)

   $f = 3e^x + \frac{1}{x} = 3e^x + x^{-1} \quad g = 1 + \frac{1}{x^2} = 1 + x^{-2}$

   $f' = 3e^x - x^{-2} \quad g' = 0 - 2x^{-3}$

   $f'(x) = (3e^x - x^{-2})(1 + x^{-2}) + (3e^x + x^{-1})(-2x^{-3})$

(e) $f(x) = \frac{\ln x}{x^2+3}$

$f = \ln x$ $\qquad g = x^2+3$

$f' = \frac{1}{x}$ $\qquad g' = 2x$

$$f'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{\frac{1}{x}\cdot(x^2+3) - \ln x \cdot 2x}{(x^2+3)^2}$$

(f) $f(x) = \frac{3-2e^x}{1-2x}$

$f = 3-2e^x$ $\qquad g = 1-2x$

$f' = -2e^x$ $\qquad g' = -2$

$$f'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{-2e^x\cdot(1-2x) - (3-2e^x)(-2)}{(1-2x)^2}$$

(g) $f(x) = \frac{x}{x+2}$

$f = x$ $\qquad g = x+2$

$f' = 1$ $\qquad g' = 1$

$$f'(x) = \frac{1\cdot(x+2) - x\cdot 1}{(x+2)^2}$$

2. If $C(x)$ is the cost of producing x items, then the **average cost**, denoted by $\bar{C}(x)$ is the cost of all the items divided by the number of items, that is,

$$\bar{C}(x) = \frac{C(x)}{x}$$

The marginal average cost is given by

$$\frac{d}{dx}\bar{C}(x) = \bar{C}'(x)$$

Use the quotient rule to calculate the marginal average cost function.

$f = C(x)$ $\qquad g = x$

$f' = C'(x)$ $\qquad g' = 1$

$$\Rightarrow \bar{C}'(x) = \frac{f'g - fg'}{g^2}$$

$$= \frac{C'(x)x - C(x)}{x^2}$$

$$= \frac{1}{x}\left(C'(x) - \frac{C(x)}{x}\right)$$

$$= \frac{1}{x}\left(C'(x) - \bar{C}(x)\right).$$

3. Given the following functions, determine $f'(x)$:

   (a) $f(x) = (2x + \pi)^7$

   $f'(x) = 7(2x+\pi)^6 \cdot 2 \quad = 14(2x+\pi)^6$

   (b) $f(x) = (2e^x - 3)^{\frac{3}{2}}$

   $f'(x) = \frac{3}{2}(2e^x - 3)^{1/2} \cdot 2e^x \quad = 3e^x(2e^x - 3)^{1/2}$

   (c) $f(x) = 6\sqrt[3]{x^2 + 1}$ $= 6(x^2+1)^{1/3}$

   $f'(x) = \frac{6}{3}(x^2+1)^{-2/3} \cdot 2x = 4x(x^2+1)^{-2/3}$

   (d) $f(x) = \frac{\ln x}{\sqrt[4]{3x^3+1}}$ (*Hint*: You will need chain rule and quotient rule.)

   $$f'(x) = \frac{\left(\frac{1}{x}\right)(3x^3+1)^{1/4} - \ln x \cdot \frac{1}{4}(3x^3+1)^{-3/4} \cdot 9x^2}{(3x^3+1)^{1/2}}$$

   (e) $f(x) = (2x - 3)^5(4x^3 + 7)$ (*Hint*: You will need chain rule and product rule.)

   $f'(x) = \left(5(2x-3)^4 \cdot 2\right)(4x^3+7) + (2x-3)^5(12x^2)$