Curve Sketching

(1) Find the vertical and horizontal asymptotes of the following functions. Whether they exist or not, make sure to justify your answers using limits. Finally, sketch a rough graph of just the asymptotes.

(a) $f(x) = \dfrac{10000x}{x^3 + x}$

Solution.

VA's:

Guess: $x^3 + x = 0 \implies x(x^2+1) = 0 \implies x = 0.$

Check:

$$\lim_{x\to 0} \frac{1000x}{x^3 + x} = \lim_{x\to 0} \frac{1000}{x^2+1} = 1000 \neq \pm\infty.$$

Thus, $x = 0$ is not a VA (it is removable) and so there are no VA's.

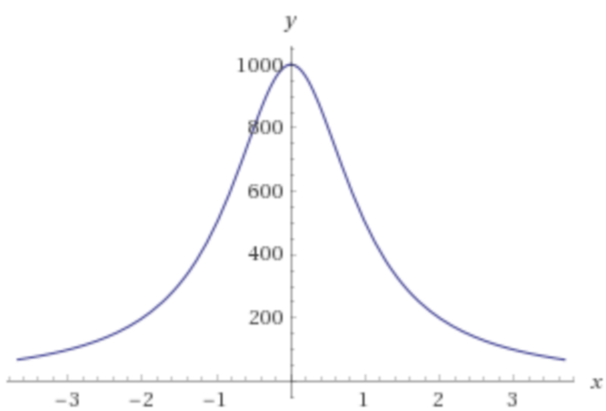
HA's:

$$\lim_{x\to\infty} \frac{1000x}{x^3 + x} = \lim_{x\to\infty} \frac{\frac{1000}{x^2}}{1 + \frac{1}{x^2}} = \frac{0}{1+0} = 0.$$

Thus, $y = 0$ is a HA.

$$\lim_{x\to-\infty} \frac{1000x}{x^3 + x} = \lim_{x\to-\infty} \frac{\frac{1000}{x^2}}{1 + \frac{1}{x^2}} = \frac{0}{1+0} = 0.$$

Thus, $y = 0$ is the *only* HA.



(b) $f(x) = \frac{9x^3+1}{7x^2}$

Solution.

VA's:

Guess: $7x^2 = 0 \implies x = 0$.

Check:

$$\lim_{x\to 0} \frac{9x^3+1}{7x^2} = \frac{1}{0}.$$

So check left and right hand limits:

$$\lim_{x\to 0^-} \frac{9x^3+1}{7x^2} = \frac{1}{\text{small} + \#} = +\infty$$

$$\lim_{x\to 0^+} \frac{9x^3+1}{7x^2} = \frac{1}{\text{small} + \#} = +\infty.$$

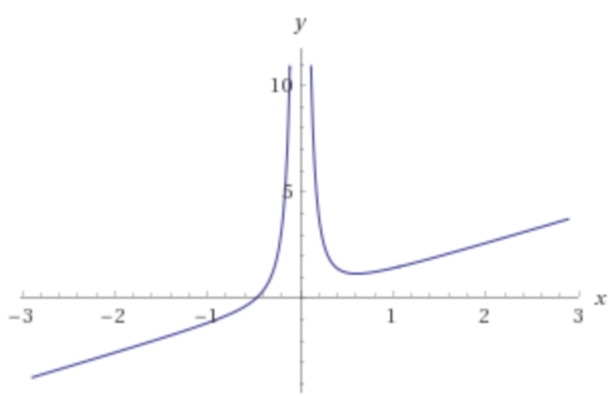
Thus, $x = 0$ is a VA.

HA's:

$$\lim_{x\to\infty} \frac{9x^3+1}{7x^2} = \lim_{x\to\infty} \frac{9x + \frac{1}{x^2}}{7} = \frac{\lim_{x\to\infty} 9x^3 + 0}{7} = \infty.$$

$$\lim_{x\to-\infty} \frac{9x^3+1}{7x^2} = \lim_{x\to-\infty} \frac{9x + \frac{1}{x^2}}{7} = \frac{\lim_{x\to-\infty} 9x^3 + 0}{7} = -\infty.$$

Thus, there are no HA's since the above limits **do not go to a finite number.**



(c) $f(x) = \frac{9x^3+1}{7x^3}$

Solution.

VA's:

Guess: $7x^3 = 0 \implies x = 0$.

Check:

$$\lim_{x\to 0} \frac{9x^3+1}{7x^3} = \frac{1}{0}.$$

So check left and right hand limits:

$$\lim_{x\to 0^-} \frac{9x^3+1}{7x^3} = \frac{1}{\text{small - } \#} = -\infty$$

$$\lim_{x\to 0^+} \frac{9x^3+1}{7x^2} = \frac{1}{\text{small} + \#} = +\infty.$$

Thus, $x = 0$ is a VA.

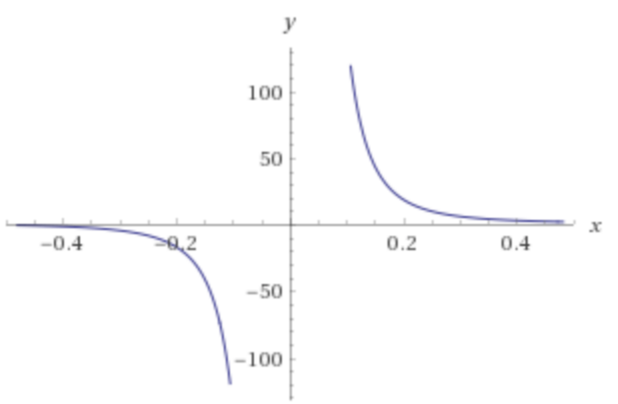
HA's:

$$\lim_{x\to\infty} \frac{9x^3+1}{7x^3} = \lim_{x\to\infty} \frac{9+\frac{1}{x^3}}{7} = \frac{9+0}{7} = \frac{9}{7}.$$

Thus, $y = \frac{9}{7}$ is a HA.

$$\lim_{x\to-\infty} \frac{9x^3+1}{7x^3} = \lim_{x\to-\infty} \frac{9+\frac{1}{x^3}}{7} = \frac{9+0}{7} = \frac{9}{7}.$$

Thus, $y = \frac{9}{7}$ is the *only* HA.



(2) Let $f(x) = x^5 \ln x$.

(a) What is the domain of f?

(b) What are the intervals of increase/decrease?

(c) Find the x-coordinate(s) of any local minima or maxima.

(d) Find the intervals of concavity.

(e) Find the $x-$coordinate(s) of any inflection points.

Consider the equation below.

$f(x) = x^5 \ln x$

(a) What is the domain of the function?

$(0, \infty)$

(b) Find the interval(s) on which f is increasing. (Enter your answer using interval notation.)

$(e^{-1/5}, \infty)$

Find the interval(s) on which f is decreasing. (Enter your answer using interval notation.)

$(0, e^{-1/5})$

(c) Find the x-coordinate(s) of any local minima. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

Local minima at x = $\frac{1}{\sqrt[5]{e}}$

Find the x-coordinate(s) of any local maxima. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

Local maxima at x = DNE

(d) Find the interval(s) on which f is concave up. (Enter your answer using interval notation.)

$(e^{-9/20}, \infty)$

Find the interval(s) on which f is concave down. (Enter your answer using interval notation.)

$(0, e^{-9/20})$

Find the x-coordinate(s) of any inflection points. (Enter your answers as a comma-separated list. If an answer does not exist, enter DNE.)

Inflection point(s) at x = $\frac{1}{e^{9/20}}$