Music + Math Summary 3

In general, the space between each sample is

$$\frac{1}{\text{sampling rate}}\, s = \frac{1}{f_s}\, s$$

Given a sound of length l, the # of samples taken will be $l \times f_s$.

The samples are then taken at the following times:

$$0, \frac{1}{f_s}, \frac{2}{f_s}, \frac{3}{f_s}, \ldots, l \times f_s - \frac{1}{f_s}$$

Ex: Suppose you sample a 20 second sound with sampling rate 10 samples/s. (a) What is the time between each sample, (b) how many samples will be taken, + (c) at what times will each sample be taken?

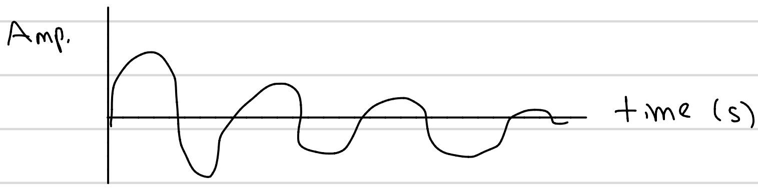
$$\text{time between each sample} = \frac{1}{f_s}\, s = \frac{1}{10}\, s.$$

$$\#\ \text{samples} = l \times f_s = 20\,s \times 10\ \text{samples/s} = 200\ \text{samples}$$

$$\begin{aligned}\text{times} &= 0, 1/10, 2/10, 3/10, \ldots, 200 - 1/10 \\ &= 0, .1, .2, .3, \ldots, 199.9\end{aligned}$$

Frequency Domain

So far everything we have done has been in what's called the time domain, i.e. time vs. amplitude



This is very useful for many things:

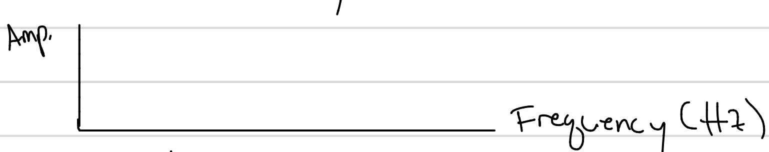
- its how the pc records digital sound
- time domain info can be used to digitally speed/slow sound down,
- delete, copy, paste sound
- add reverb, echo, etc.
- increase/decrease volume of entire sound (level)

However, with only time info how could we:

- pitch correct/auto tune
- increase/decrease volume of specific frequencies (EQ - add bass, etc.)
- analyze frequency data
- remove certain frequencies (low/high pass filters)

* to do all of these things digitally, we need info about the frequencies

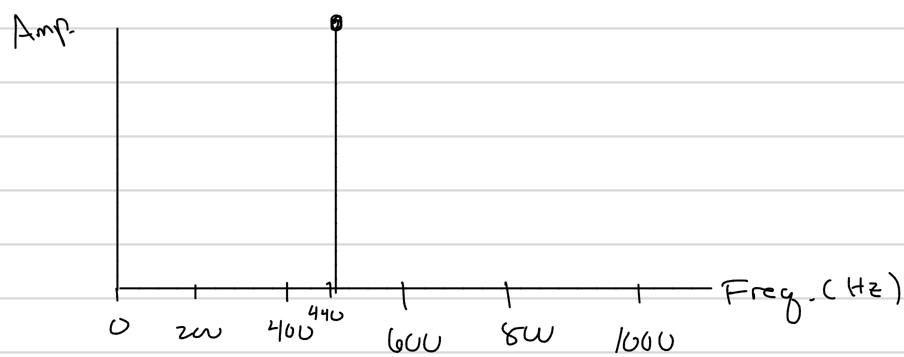
This called the frequency domain (also called spectrum)



LABEL

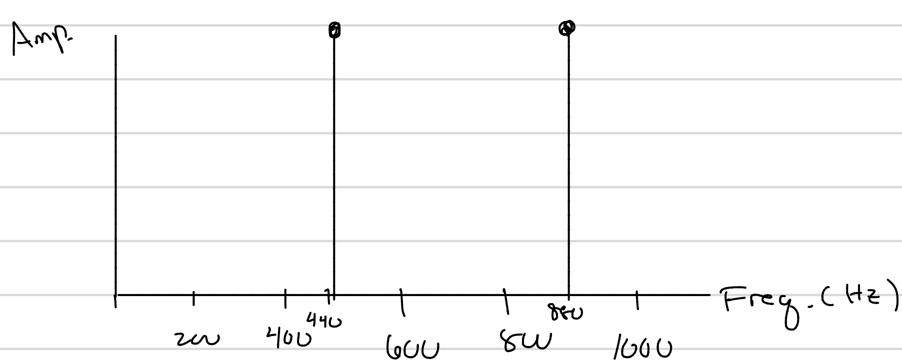
Recall: $S(t) = \sin(2\pi \times 440t)$ represents a pure tone w/ frequency 440 Hz.

Since this is a pure tone (constant freq.) we would expect the spectrum to only have one spike at 440 Hz and be 0 everywhere else:



What would we expect for

$$S(t) = \sin(2\pi \times 440t) + \sin(2\pi \times 880t)?$$



So how exactly do we get from time domain to freq. domain?

* the idea is simple but the implementation is hard!

Idea: We know by superposition that all sounds are really just sums of diff. pure tones. So the idea is to somehow take a sound and decompose it into its pure tunes since we can easily identify the frequency of each pure tone.

- Once we know all the pure tunes, we know all the frequencies in the whole sound!
- the hard part is how do we decompose a sound into its pure tones?

* The way this is done is with a mathematical tool called the Fast Fourier Transform (FFT).