Exam 1

Great Ideas in Mathematics, MAT 110

October 16, 2017

Name: SOLUTIONS

Directions: Complete each problem**.** In order to maximize your score, make sure to show all of your work. Even without a final (or correct) answer, you can still get plenty of partial credit!

1. We discussed four voting methods: Plurality, Borda Count, Plurality-with-Elimination, and Pairwise Comparison.

   (a) Give a brief description of the idea behind each method. [8]

   (1) Plurality - Most # of first place votes wins
   (2) Borda - each place is assigned points; candidate w/ most total points wins
   (3) Plurality w/ elimination - Lowest candidate is eliminated & votes are transferred to next eligible, repeat until there is a majority winner
   (4) P.C. - Each candidate is compared w/ every other candidate

   (b) Pick two methods and list the pros and cons. [4]

   (c) What did Arrow's Impossibility Theorem say? [3]

   No voting method meets every one of his fairness criteria

2. The Brute-Force Algorithm:

(a) Briefly describe the Brute-Force Algorithm. [6]

List all Hamilton circuits & their costs. Optimal tour is the one w/ the least cost.

(b) List one pro and one con of the Brute-Force Algorithm. [4]

Pro: Always finds optimal tour

Con: Computationally expensive

3. The Nearest-Neighbor Algorithm:

(a) Briefly describe the Nearest-Neighbor Algorithm. [6]

At each location you move to the next location by choosing the edge w/ the least cost.

(b) List one pro and one con of the Nearest-Neighbor Algorithm. [4]

Pro: Fast & efficient / finds a "good" tour

Con: Doesn't always find optimal tour

4. You are planning to visit four cities, *A*, *B*, *C*, and *D*. The following table shows the time (in hours) that it takes to travel by car between any two cities. Using the **Brute-Force Algorithm**, find an **optimal tour** and the **optimal cost** for this TSP that **starts and ends at *B*.** [15]

	A	*B*	*C*	*D*
A	*	12	6	14
B	12	*	17	15
C	6	17	*	11
D	14	15	11	*

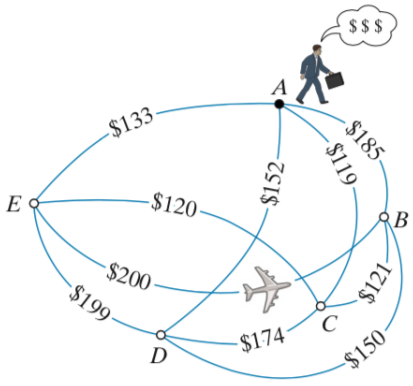
4 sites = 6 Hamilton circuits starting/ending at B

B, A, C, D, B = 12 + 6 + 11 + 15 = 44 B, D, C, A, B

B, C, A, D, B = 17 + 6 + 14 + 15 = 52 B, D, A, C, B

B, C, D, A, B = 17 + 11 + 14 + 12 = 54 B, A, D, C, B

5. For the weighted graph shown in the following figure, find the **Nearest-Neighbor tour** with **starting vertex *B*** and calculate the **total cost** of this tour. [15]



B, C, A, E, D, B

= 121 + 119 + 133 + 199 + 150

6. The city of Danbury currently has 137 streetlights (a completely false fact). The city council has decided to install and have operational 2 additional streetlights at the end of each week for the next 52 weeks.

   (a) What is the linear growth formula that models how many streetlights the city will have at the end of N weeks? [Hint: Recall the linear growth formula $P_N = P_0 + rN$.] [10]

$r = 2$, $P_0 = 137$ $\Rightarrow$ $P_N = 137 + 2N$

   (b) How many streetlights will the city have at the end of the 52 weeks? [5]

$P_{52} = 137 + 2(52) = 137 + 104 = 241$

7. Assume that the population of fish in a pond grows according to an exponential growth model. The initial population of fish in the pond is $P_0 = 10$. After one month there are 15 fish in the pond.

   (a) What is the growth rate of fish within the first month? [Hint: Recall that the growth rate for exponential growth is $r = \frac{Y-X}{X}$.] [5]

$r = \frac{15-10}{10} = \frac{5}{10} = \frac{1}{2}$

   (b) What is the exponential growth formula that models how many fish are in the pond at the end of N months? [Hint: Recall the exponential growth formula $P_N = (1+r)^N P_0$, where r is the growth rate.] [10]

$P_0 = 10$

$P_N = (1 + \frac{1}{2})^N 10 = \frac{3}{2}^N \times 10$

   (c) How many fish are in the pond after 10 months? [5]

$P_{10} = \frac{3}{2}^{10} \times 10 =$