# Week 4 Lecture 2

September 20, 2017

- **Stretch 17 (Pg. 99):** Which is bigger, 100000001/100000002 or 200000001/200000002?
  - **Hints:** Notice that both numbers look like $n/(n+1)$.
  - **Solution:** When $n = 0$, it's 0. When $n = 1$ it's 1/2. When $n = 2$ it's 2/3. When $n = 3$, it's 3/4. As $n$ increases, $n/(n+1)$ increases. Since 200000001 is greater than 100000001, 200000001/200000002 is bigger.
  - **Alternate Solution:** The first fraction is shy of 1 by just 1/100000002. The second fraction is shy of 1 by just 1/200000002, making is larger since it is shy of 1 by a smaller value.
- **Ch. 5: Simplify It (Section: Try Simple Cases and Look for a Pattern)**
  - **Changing Places**
    - Break into groups of 3-4 and use the game board in the text (11 squares, 5 on the left, the center unoccupied, and five on the right). Students can use coins or any other convenient token. The rules are as follows: The tokens that started out on the right can only move towards the left and vice versa. Each turn consists of a token moving 1 space into an unoccupied square or by jumping another piece into an unoccupied square. You cannot move two spaces or jump over two men. The goal is for all the tokens to migrate to the other side and change places
    - After continuing to get stuck, pause and reflect on how to recognize what gets you gridlocked.
    - Come up with a way to simplify the game.
    - Try the game with just 1 square on either side of the central blank square. You should finish the game in 3 moves.
    - Now try it with two squares on each side. Most should finish in 8 moves.
    - Build a table keeping track of your progress and proceed up to the 5 squares on each side.
    - If it's still too hard, pause and reflect on trying to notice a pattern. Notice that as we went from one to two spaces on a side, the number of moves increased by 5. When we progressed from two spaces to three, the number of moves increased by 7. If this pattern continues, we should expect the next jump to be 9 moves, which when added to 15 moves should yield 24. Try this out to confirm.

| Squares on 1 side | # of moves |
|---|---|
| 1 | 3 |
| 2 | 8 |
| 3 | 15 |
| 4 | 24 |
| 5 | 35 |

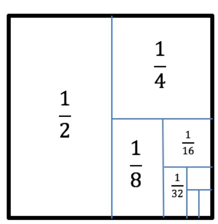
* Now, using the pattern, how many moves will 5 squares require? $24 + 11 = 35$ moves
* We can figure out exactly which moves will win the game by noticing a pattern. The pattern of moves takes on a predictable, symmetric nature and makes executing the five-square scenario very easy:

MJM
MJMJJMJM
MJMJJMJJJMJJMJM
MJMJJMJJJMJJJJMJJJMJJMJ
MJMJJMJJJMJJJJMJJJJJMJJJJMJJJMJJMJM

– **Telescoping Sums (if time):** What is the sum of the following series

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \cdots + \frac{1}{100 \times 101}?$$

* **Hints:** It would be very time consuming to add up all of the 100 fractions.
* Build a table and start calculating the partial sums, looking for a pattern.
* The pattern is that *the sum of the first $n$ fractions is* $\frac{n}{n+1}$.
* **Solution:** It seems reasonable to conjecture that the sum of the first 100 fractions will be $100/101$.
* **What if the sum didn't stop at 100 and you had to add all these numbers up forever?**
  · Do you think the sum will be a number (converge) or if it will be infinity (diverge)? Is it possible to add up infinitely many numbers and get a finite number?
  · Start with a $1 \times 1$ square (area is 1):

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  · Notice that the area of the box is $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$. This is an infinite sum of positive fractions that equals the area of the box which clearly equals 1!
  · Recall from above that the sum of the first $n$ fractions is $\frac{n}{n+1}$. In Calculus, we can use "limits" to show

$$\lim_{n \to \infty} \frac{n}{n+1} = \lim_{n \to \infty} \frac{1}{1 + \frac{1}{n}} = 1.$$

That is, the sum of all infinitely many fractions is 1!

- **Homework: Parking Lots [Pg. 56] (Due 9/27/17)**
  - First, simplify the problem by supposing there is only one space
  - Make assumptions:
    - Assume that shoppers only shop for 1 hour in the mall before they leave (does this seem reasonable?)
    - Assume that the arrival of shoppers is uniform
  - So how long do we expect to wait for one space to open up?
    - We could either wait for one second if the owner came out immediately, or we could wait one hour (on average) if the owner has just gone into the mall
    - Seems logical to guess that the space will become available somewhere between these two extremes - Might as well guess in the middle: 1/2 hour
    - So for one space to open we expect to wait 30 minutes
  - How about two spaces?
    - Since we assumed that the arrival of shoppers is uniform, this would mean that one shopper arrived 20 minutes ago and the other arrived 40 minutes ago. [Draw this on a number line to *see* the uniform assumption - will make it easier when you increase the number of spaces.]
    - Hence, we expect to wait 20 minutes for a space to open up.