Derivatives Cheat Sheet

Derivative Rules

1. Constant Rule: $\frac{d}{dx}(c) = 0$, where c is a constant
2. Power Rule: $\frac{d}{dx}(x^n) = nx^{n-1}$
3. Product Rule: $(fg)' = f'g + fg'$
4. Quotient Rule: $\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}$
5. Chain Rule: $(f(g(x))' = f'(g(x))g'(x)$

Common Derivatives

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \qquad \frac{d}{dx}(\cos x) = -\sin x \qquad \frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x \qquad \frac{d}{dx}(\csc x) = -\csc x \cot x \qquad \frac{d}{dx}(\cot x) = -\csc^2 x$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

Exponential & Logarithmic Functions

$$\frac{d}{dx}(a^x) = a^x \ln(a) \qquad \frac{d}{dx}(e^x) = e^x$$

$$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)} \qquad \frac{d}{dx}(\ln(x)) = \frac{1}{x}$$

Chain Rule

In the below, $u = f(x)$ is a function of x. These rules are all generalizations of the above rules using the chain rule.

1. $\frac{d}{dx}(u^n) = nu^{n-1}\frac{du}{dx}$
2. $\frac{d}{dx}(a^u) = a^u \ln(a)\frac{du}{dx}$
3. $\frac{d}{dx}(e^u) = e^u\frac{du}{dx}$
4. $\frac{d}{dx}(\log_a(u)) = \frac{1}{x\ln(u)}\frac{du}{dx}$
5. $\frac{d}{dx}(\ln(u)) = \frac{1}{u}\frac{du}{dx}$
6. $\frac{d}{dx}(\sin(u)) = \cos(u)\frac{du}{dx}$
7. $\frac{d}{dx}(\cos(u)) = -\sin(u)\frac{du}{dx}$
8. $\frac{d}{dx}(\tan(u)) = \sec^2(u)\frac{du}{dx}$
9. **Same idea for all other trig functions**
10. $\frac{d}{dx}(\tan^{-1}(u)) = \frac{1}{1+u^2}\frac{du}{dx}$
11. **Same idea for all other inverse trig functions**

Implicit Differentiation

Use whenever you need to take the derivative of a function that is **implicitly** defined (not solved for y). Examples of implicit functions: $\ln(y) = x^2$, $x^3 + y^2 = 5$, $6xy = 6x + 2y^2$, etc.

Implicit Differentiation Steps:

1. Differentiate both sides of the equation with respect to "x"
2. When taking the derivative of any term that has a "y" in it multiply the term by y' (or dy/dx)
3. Solve for y'

When finding the second derivative y'', remember to replace any y' terms in your final answer with the equation for y' you already found. In other words, your final answer should **not** have any y' terms in it.

Log Differentiation

Two cases when this method is used:

- **Use whenever you can take advantage of log laws to make a hard problem easier**
  - Examples: $\dfrac{(x^3+x)\cos x}{x^2+1}$, $\ln(x^2+1)\cos(x)\tan^{-1}(x)$, etc.
  - Note that in the above examples, log differentiation is **not required** but makes taking the derivative easier (allows you to avoid using multiple product and quotient rules)
- **Use whenever you are trying to differentiate**

$$\frac{d}{dx}\left(f(x)^{g(x)}\right)$$

  - Examples: x^x, $x^{\sqrt{x}}$, $(x^2+1)^x$, etc.
  - Note that in the above examples, log differentiation is **required**. There is no other way to take these derivatives.

Log Differentiation Steps:

1. Take the ln of both sides
2. Simplify the problem using log laws
3. Take the derivative of both sides (implicit differentiation)
4. Solve for y'